## A novel Bayesian model for assessing intratumor heterogeneity of tumor infiltrating leukocytes with multi-region gene expression sequencing

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Introduction
Intratumor heterogeneity (ITH) of tumor-infiltrated leukocytes (TILs) is an important phenomenon of cancer biology will portuity that allows for explorations of TILs and their ITH for each subiect described as follows:


Figure 1. Multi-region sequencing study design
ICelTH model is designed to address the challenges of assessing ITH using multi-region RNA-sed data, as they can reveal differences in gene expression and immune cell infiltration between
different regions of the same tumor.

## Method

Considering a total of $K$ immune cell types are of interest here, the observed gene expression $Y_{s q}$ can be decomposed as

$$
\begin{equation*}
\log \left(Y_{s g}\right)=\sum_{k} h_{s k} W_{s g k}+\epsilon_{s g}, \text { for } s \in I_{i}, \tag{1}
\end{equation*}
$$

where $h_{s k}$ is the unobserved cellular abundance from cell type $k$ in sample $s$, and $W_{s g k}$ is a hree-dimensional tensor that stands for the hidden expression profiles of gene $g$ in sample $s$ from cell type $k$. $\epsilon_{s g}$ is the error term that follows a normal distribution.
To characterize intratumor heterogeneity within the same patient subject, a hierarchical Bayesian o characterize intratumor heterogeneity within the same patient subject, a hierarchical Bayesian profile parameters $\mu_{i g k}$ per gene and per cell type. This is expressed as:

$$
\log \left(W_{s g k}\right) \stackrel{\text { i.i.d }}{\sim} N\left(\mu_{i g k}, \frac{1}{\lambda_{g k}}\right) \text {, for } s \in I_{i} \text {. }
$$

Second, we use Dirichlet distribution to model cell-type-specific parameter, $h_{\text {sk }}$. For sample $s \in I_{i}$, we have

$$
h_{s 1}, \ldots, h_{s K} \sim \operatorname{Dir}\left(C_{i} \pi\right),
$$

where (1) $\boldsymbol{\pi}$ is a $K$ by 1 vector pooled across all samples with $\sum_{k} \pi_{k}=1$ to represent the global cellular composition, and (2) $C_{i}$ is a patient-specific parameter that controls the variability of the ellular composition across samples within each patient, thereby revealing ITH
The Fenton-Wilkinson (FW) approximation [1] is used to approximate equation (1) by another g-normal distribution as fillow:

$$
\log \left(Y_{s g}\right)=N\left(\log \left(\sum_{k} h_{s k} W_{s g k}\right), \frac{1}{\lambda_{s g}}\right) \text {, for } s \in I_{i}, i=1, \ldots, n .
$$

Prior specifications
To incorporate prior knowledge from existing reference profiles, we use conjugate prior distributions for the patient-specific mean expression parameter $\mu_{i g k}$ 's and variability $\lambda_{g k}$ :

$$
\mu_{i g k} \stackrel{\text { i.i.d }}{\sim} N\left(\mu_{g k}, \frac{1}{\rho_{g k} \lambda_{g k}}\right), \quad \lambda_{g k} \sim \operatorname{Gamma}\left(\alpha_{g k}, \beta_{g k}\right),
$$

where $\mu_{g k}, \alpha_{g, k}$ and $\beta_{g, k}$ are determined by the mean and variance from the reference matrix. Optimization
We use the Collapsed Variational Bayesian (CVB) method to optimize the ICelTH model. To perform a CVB method, we first marginalize over the hidden random variables $\mu_{i g k}{ }^{\prime}$ 's and $\lambda_{g k}$ 's,

$$
\prod_{i=1}^{N} \prod_{\leqslant I} P\left(\log \left(W_{s g k}\right) \mid \mu_{g k}, \rho_{g k}, \alpha_{g k}, \beta_{g k}\right)
$$

$$
=\int_{\lambda} \int_{\mu_{i=1}} \prod_{1 \in I_{i}}^{N} \prod p\left(\log \left(W_{s g k}\right) \mid \mu_{i g k}, \lambda_{g k}\right) \times p\left(\mu_{i g k} \mid \mu_{g k}, \lambda_{g k}, \rho_{g k}\right) \times p\left(\lambda_{g k} \mid \alpha_{g k}, \beta_{g k}\right) d \mu_{1 g k} \cdots d \mu_{N g k} d \lambda_{g k}
$$ the remaining unobserved variables, that is $W_{s g k}$ and $h_{s k}$

$$
Q\left(\log \left(W_{s g k}\right)\right) \sim N\left(\gamma_{i g k}, \tau_{g k}^{2}\right), \quad Q\left(h_{s 1}, \ldots, h_{s K}\right) \sim \operatorname{Dir}\left(\xi_{s 1}, \ldots, \xi_{s K}\right),
$$

where $Q(\cdot)$ denotes the variational distribution aims to approximate the true posterior density and $\{\tau$. $\left.)()^{2}\right)$ there are $(N \times G \times K)+(G \times K)+\left(\sum_{i=1}^{N} I_{i} \times K\right)$ parameters to estimate,
The final objective function is based on the evidence lower bound, derived as follows:
$\log (P(Y)) \geq E_{Q(W, H)}\left\{\log \frac{P(Y, W, H \mid \boldsymbol{\theta})}{Q(W H)}\right\}$
where $Z=(W, H)$ and $\boldsymbol{\theta}=(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\rho}, \boldsymbol{\mu}, \boldsymbol{\lambda})$ denote the unobserved variables and hyperparameters, respectively. We apply Limited-memory BFGS (Broyden-Fletcher-Goldfarb-Shanno) to iteratively maximize the objective function defined in equation 3 and its gradient with respect to variational parameters has been derived to speed the optimization.
To obtain the intratumor heterogeneity parameter $C_{i}$ in equation 2 for a specific subject $i$, we compute the first and second central moment for $s \in I_{i}$ as follows,

$$
\begin{equation*}
E\left[h_{s, k}\right]=\frac{C_{i} \hat{त}_{k}}{C_{i} \sum_{c} \hat{\pi}_{c}}=\hat{\pi}_{k}, \quad \operatorname{Var}\left[h_{s, k}\right]=\frac{\hat{\pi}_{k}\left(1-\hat{\pi}_{k}\right)}{C_{i}+1} . \tag{4}
\end{equation*}
$$

Then, the total variance across samples within subject $i$ is $\sum_{k} \operatorname{Var}\left[h_{s, k}\right]=\sum_{k} \frac{\hat{\tau}_{k}\left(1-\hat{\pi}_{k}\right)}{C}$, which the ITH score can be calculated as follow

$$
\begin{equation*}
\hat{C}_{i}=f_{C}\left(\hat{\pi}_{k}, \hat{h}_{s k}\right)=\frac{\sum_{k} \hat{\pi}_{k}\left(1-\hat{\pi}_{k}\right)}{\sum_{i v} \operatorname{var}\left(h_{s k}\right)}-1 . \tag{5}
\end{equation*}
$$

This equation estimates the degree of ITH within each patient. In particular, the numerator in equation 5 represents the expected variance of the cell type proportions across all samples, while the denominator represents the actual variance within each subject.

$$
\begin{aligned}
& \geq \underbrace{E_{Q}\{\log P(Y \mid W, H, \lambda)\}}+\underbrace{E_{Q}\{\log P(W \mid \mu, \rho, \alpha, \beta)\}}+\underbrace{E_{Q}\{\log P(H \mid C, \pi)\}} \text { (3) } \\
& \underbrace{E_{Q}\{\log P(W \mid \gamma, \tau)\}}-\underbrace{E_{Q}\{\log P(H \mid \xi)\}}_{e}
\end{aligned}
$$

Simulation study
We conduct extensive simulation studies to evaluate the performance and robustness of our We conduct extensive simulation stuaties to evaluate the performance and robustness of


Figure 2 . Results of analyzing the simulation data in the setting with four cell types and randomly generated sample The results show the ICeITH provides the most accurate estimation of cell-type-specific proportions and a better ability to dichotomize low vs high intratumor heterogeneity groups. Real data application on TRACERx

We apply the ICelTH model to analyze RNA-seq data in patients from the TRACERx cohort [2]. The multi-region RNA-seq data are available in 45 patients, and it results in 140 tumors in total. We seek to associate the survival outcome with intratumor heterogeneity scores estimated from our proposed method.


Figure 3 . Results of analyzing the TRACERx data.
CelTH is capable of classifying patients into different risk group according to the ITH estimation CelTH is capable of classifying patients into different risk groups acco

References

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